Comprehension Model Equations

1 Terms

Boolean fork and join variables $f, j \in \mathbb{B} = \{\top, \bot\}$

Words $w \in \mathbb{W}$

Category labels $\ell \in \mathbb{L}$

Semantic contexts $k \in \mathbb{K}$ (e.g. eat_2, $apple_1$ for an eaten apple; $eat_0, apple_{1,-2}$ for eating an apple)

Context sets $K, K' \in \mathbb{B}^{\mathbb{K}}$ (a.k.a. $\mathcal{P}(\mathbb{K})$, a.k.a. $2^{\mathbb{K}}$)

Operators $o, o' \in \mathbb{O} \subset \mathbb{B}^{\mathbb{K}} \rightarrow \mathbb{B}^{\mathbb{K}}$

Signs $a, b, c, p = K : \ell = \langle K, \ell \rangle \in \mathbb{B}^{\mathbb{K}} \times \mathbb{L}$

Incomplete signs $q = a/b = \langle a, b \rangle \in (\mathbb{B}^{\mathbb{K}} \times \mathbb{L}) \times (\mathbb{B}^{\mathbb{K}} \times \mathbb{L})$

Depth $d \in \{1, ..., D\}$

2 Model equations

$$
\begin{aligned}
\mathsf{P}(q_t^{1..D}\; w_t \,|\, q_{1..t\text{-}1}^{1..D}\; w_{1..t\text{-}1}) &= \mathsf{P}(q_t^{1..D}\; w_t \,|\, q_{t\text{-}1}^{1..D}) && \text{Markov assumption} \\
&\overset{\text{def}}{=} \mathsf{P}(f_t\; K_{p_t}\; \ell_{p_t}\; w_t\; j_t\; o_t\; o'_t\; a_t^{1..D}\; b_t^{1..D} \,|\, q_{t\text{-}1}^{1..D}) && \text{redundant variables} \\
&= \mathsf{P}_\phi(f_t\; K_{p_t} \,|\, q_{t\text{-}1}^{1..D}) \cdot && \text{fork, preterminal context} \\
&\quad \mathsf{P}_\pi(\ell_{p_t} \,|\, q_{t\text{-}1}^{1..D}\; f_t\; K_{p_t}) \cdot && \text{preterminal label} \\
&\quad \mathsf{P}_\xi(w_t \,|\, q_{t\text{-}1}^{1..D}\; f_t\; K_{p_t}\; \ell_{p_t}) \cdot && \text{word} \\
&\quad \mathsf{P}_\iota(j_t\; o_t\; o'_t \,|\, q_{t\text{-}1}^{1..D}\; f_t\; K_{p_t}\; \ell_{p_t}\; w_t) \cdot && \text{join, left/right operators} \\
&\quad \mathsf{P}_\alpha(a_t^{1..D} \,|\, q_{t\text{-}1}^{1..D}\; f_t\; K_{p_t}\; \ell_{p_t}\; w_t\; j_t\; o_t\; o'_t) \cdot && \text{apex signs} \\
&\quad \mathsf{P}_\beta(b_t^{1..D} \,|\, q_{t\text{-}1}^{1..D}\; f_t\; K_{p_t}\; \ell_{p_t}\; w_t\; j_t\; o_t\; o'_t\; a_t^{1..D}) && \text{brink signs}
\end{aligned}
$$

Fork decision and preterminal context:

$$
\mathsf{P}_\phi(f_t\; K_{p_t} \,|\, q_{t\text{-}1}^{1..D}) \overset{\text{def}}{=} \mathsf{P}_\phi(f_t\; K_{p_t} \,|\, d\; K_{b_{t\text{-}1}^d}); \quad d = \max_{d'}\{q_{t\text{-}1}^{d'} \neq q_\bot\}
$$

$$
\mathsf{P}_\phi(f\; K' \,|\, d\; K) \overset{\text{def}}{=} \frac{\exp \sum_{k \in K, k' \in K'} \phi_{d,k,f,k'}}{\sum_{f,K'} \exp \sum_{k \in K, k' \in K'} \phi_{d,k,f,k'}}
$$

Preterminal label:

$$
\mathsf{P}_\pi(\ell_{p_t} \,|\, q_{t\text{-}1}^{1..D}\; f_t\; K_{p_t}) \overset{\text{def}}{=} \mathsf{P}_\pi(\ell_{p_t} \,|\, d\; b_t^d\; K_{p_t}); \quad d = \max_{d'}\{q_{t\text{-}1}^{d'} \neq q_\bot\}
$$

Word:

$$\mathsf{P}_\xi(w_t \,|\, q^{1..D}_{t\text{-}1}\ f_t\ K_{p_t}\ \ell_{p_t}) \stackrel{\text{def}}{=} \mathsf{P}_\xi(w_t \,|\, K_{p_t}\ \ell_{p_t})$$

Join decision and left/right operators:

$$\mathsf{P}_\iota(j_t\ o_t\ o'_t \,|\, q^{1..D}_{t\text{-}1}\ f_t\ K_{p_t}) \stackrel{\text{def}}{=} \begin{cases} \mathsf{P}_\iota(j_t\ o_t\ o'_t \,|\, d\ K_{a^d_{t\text{-}1}}\ K_{b^{d\text{-}1}_{t\text{-}1}}); & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \bot \\ \mathsf{P}_\iota(j_t\ o_t\ o'_t \,|\, d\ K_{p_t}\ K_{b^d_{t\text{-}1}}); & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \top \end{cases}$$

$$\mathsf{P}_\iota(j\ o\ o' \,|\, d\ K\ K') \stackrel{\text{def}}{=} \frac{\exp \sum_{k\in K, k'\in K'} \iota_{d,k,k',j,o,o'}}{\sum_{j,o,o'} \exp \sum_{k\in K, k'\in K'} \iota_{d,k,k',j,o,o'}}$$

Apex signs:

$$\mathsf{P}_\alpha(a^{1..D}_t \,|\, q^{1..D}_{t\text{-}1}\ f_t\ K_{p_t}\ \ell_{p_t}\ w_t\ j_t\ o_t\ o'_t) \stackrel{\text{def}}{=}$$

$$\begin{cases} \prod_{d'=1}^{d\text{-}2} [\![a^{d'}_t = a^{d'}_{t\text{-}1}]\!] \cdot [\![a^{d\text{-}1}_t = a^{d\text{-}1}_{t\text{-}1}]\!] \cdot \prod_{d'=d}^{D} [\![a^{d'}_t = a_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \bot, j_t = \top \\ \prod_{d'=1}^{d\text{-}1} [\![a^{d'}_t = a^{d'}_{t\text{-}1}]\!] \cdot \mathsf{P}_\alpha(a^d_t \,|\, d\ o_t\ b^{d\text{-}1}_{t\text{-}1}\ a^d_{t\text{-}1}) \cdot \prod_{d'=d+1}^{D} [\![a^{d'}_t = a_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \bot, j_t = \bot \\ \prod_{d'=1}^{d\text{-}1} [\![a^{d'}_t = a^{d'}_{t\text{-}1}]\!] \cdot [\![a^d_t = a^d_{t\text{-}1}]\!] \cdot \prod_{d'=d+1}^{D} [\![a^{d'}_t = a_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \top, j_t = \top \\ \prod_{d'=1}^{d} [\![a^{d'}_t = a^{d'}_{t\text{-}1}]\!] \cdot \mathsf{P}_\alpha(a^{d+1}_t \,|\, d\ o_t\ b^d_{t\text{-}1}\ p_t) \cdot \prod_{d'=d+2}^{D} [\![a^{d'}_t = a_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \top, j_t = \bot \end{cases}$$

$$\mathsf{P}_\alpha(a \,|\, d\ o\ b\ c) \stackrel{\text{def}}{=} [\![K_a = o(K_c)]\!] \cdot \mathsf{P}_\alpha(\ell_a \,|\, d\ o\ \ell_b\ \ell_c)$$

Brink signs:

$$\mathsf{P}_\beta(b^{1..D}_t \,|\, q^{1..D}_{t\text{-}1}\ f_t\ K_{p_t}\ \ell_{p_t}\ w_t\ j_t\ o_t\ o'_t\ a^{1..D}_t) \stackrel{\text{def}}{=}$$

$$\begin{cases} \prod_{d'=1}^{d\text{-}2} [\![b^{d'}_t = b^{d'}_{t\text{-}1}]\!] \cdot \mathsf{P}_\beta(b^{d\text{-}1}_t \,|\, d\ o_t\ o'_t\ b^{d\text{-}1}_{t\text{-}1}\ a^{d\text{-}1}_{t\text{-}1}) \cdot \prod_{d'=d}^{D} [\![b^{d'}_t = b_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \bot, j_t = \top \\ \prod_{d'=1}^{d\text{-}1} [\![b^{d'}_t = b^{d'}_{t\text{-}1}]\!] \cdot \mathsf{P}_\beta(b^d_t \,|\, d\ o_t\ o'_t\ a^d_t\ a^d_{t\text{-}1}) \cdot \prod_{d'=d+1}^{D} [\![b^{d'}_t = b_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \bot, j_t = \bot \\ \prod_{d'=1}^{d\text{-}1} [\![b^{d'}_t = b^{d'}_{t\text{-}1}]\!] \cdot \mathsf{P}_\beta(b^d_t \,|\, d\ o_t\ o'_t\ b^d_{t\text{-}1}\ p_t) \cdot \prod_{d'=d+1}^{D} [\![b^{d'}_t = b_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \top, j_t = \top \\ \prod_{d'=1}^{d} [\![b^{d'}_t = b^{d'}_{t\text{-}1}]\!] \cdot \mathsf{P}_\beta(b^{d+1}_t \,|\, d\ o_t\ o'_t\ a^{d+1}_t\ p_t) \cdot \prod_{d'=d+2}^{D} [\![b^{d'}_t = b_\bot]\!]; & d = \max_{d'}\{q^{d'}_{t\text{-}1} \neq q_\bot\} \quad \text{if } f_t = \top, j_t = \bot \end{cases}$$

$$\mathsf{P}_\beta(b \,|\, d\ o\ o'\ c\ a) \stackrel{\text{def}}{=} [\![K_b = o'(K_c)]\!] \cdot \mathsf{P}_\beta(\ell_b \,|\, d\ o\ o'\ \ell_c\ \ell_a)$$